

(c)  
 (ii) 
$$\text{Ker}(\phi) = \left\{ \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \mid \frac{\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} - \begin{pmatrix} a & d & g \\ b & e & h \\ c & f & i \end{pmatrix}}{2} = \mathbf{0} \right\}$$

$$= \left\{ \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \mid \begin{pmatrix} 0 & b-d & c-g \\ d-b & 0 & f-h \\ g-c & h-f & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right\}$$

$$= \left\{ \begin{pmatrix} a & b & c \\ b & e & f \\ c & f & i \end{pmatrix} \mid a, b, c, e, f, i \in \mathbb{R} \right\}$$

$$\text{Ker}(\psi) = \left\{ \begin{pmatrix} 0 & b & c \\ -b & 0 & f \\ -c & -f & 0 \end{pmatrix} \mid b, c, f \in \mathbb{R} \right\}$$

(iii) 
$$\text{Im}(\phi) = \left\{ \begin{pmatrix} 0 & b-d & c-g \\ d-b & 0 & f-h \\ g-c & h-f & 0 \end{pmatrix} \mid b, c, d, f, g, h \in \mathbb{R} \right\}$$

$$= \left\{ \begin{pmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{pmatrix} \mid a, b, c \in \mathbb{R} \right\}$$

$$\text{Im}(\psi) = \left\{ \begin{pmatrix} 2a & b+d & c+g \\ b+d & 2e & f+h \\ c+g & f+h & 2i \end{pmatrix} \mid a, b, c, d, e, f, g, h, i \in \mathbb{R} \right\}$$

$$= \left\{ \begin{pmatrix} a & b & c \\ b & d & e \\ c & e & f \end{pmatrix} \mid a, b, c, d, e, f \in \mathbb{R} \right\}$$