

$$\det(D_n) = (-1)^n \cdot 4$$

漸化式を解くには

(*)
 $(x)_{n+1} - 4(x)_n$
 $(x)_{n+1} = 4(x)_n$

$$= -2 \det(D_n) - \det(D_{n-1})$$

$$= -2 \det \begin{pmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & D_n \end{pmatrix} - \det \begin{pmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & D_{n-1} \end{pmatrix}$$

(b) $n > 4$ とき

$$\det(D_{n+1}) = \begin{vmatrix} 2 & 1 & 0 & \dots & 0 \\ 0 & 1 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \dots & D_n \end{vmatrix} = \det(D_{n+1} \text{ 変形})$$

$$\det(D_5) = \begin{vmatrix} -2 & 1 & 0 & 0 & 0 \\ 1 & -2 & 1 & 0 & 0 \\ 0 & 1 & -2 & 1 & 0 \\ 0 & 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 1 & -2 \end{vmatrix} = -4$$

(a) $\det(D_4) = \begin{vmatrix} -2 & 1 & 0 & 0 \\ 1 & -2 & 1 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 1 & -2 \end{vmatrix} = 4$